# SAMPLE QUESTION PAPER

## CLASS-XII (2016-17)

## MATHEMATICS (041)

Time allowed: **3** hours | Maximum Marks: **100**

### General Instructions:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

## SECTION-A

**Questions from 1 to 4 are of 1 mark each.**

**1.** What is the principal value of $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$?

**2.** A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|3AB|$

**3.** What is the distance of the point (p, q, r) from the x-axis?

**4.** Let f : R → R be defined by $f(x) = 3x^2 - 5$ and g : R → R be defined by $g(x) = \frac{x}{x^2+1}$. Find $gof$

## SECTION-B

**Questions from 5 to 12 are of 2 marks each.**

**5.** How many equivalence relations on the set {1,2,3} containing (1,2) and (2,1) are there in all ? Justify your answer.

**6.** Let $l_i, m_i, n_i$ ; i = 1, 2, 3 be the direction cosines of three mutually perpendicular vectors in space. Show that $AA' = I_3$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

**7.** If $e^y(x + 1) = 1$, show that $\frac{dy}{dx} = -e^y$

**8.** Find the sum of the order and the degree of the following differential equations:

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1 + x) = 0$$

**9.** Find the Cartesian and Vector equations of the line which passes through the point (–2, 4,–5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$

**10.** Solve the following Linear Programming Problem graphically:

Maximize $Z = 3x + 4y$

subject to

$$x + y \leq 4, x \geq 0 \text{ and } y \geq 0$$

**11.** A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.

**12.** The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

**SECTION-C**

**Questions from 13 to 23 are of 4 marks each.**

**13.** If A + B + C = $\pi$, then find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

**OR**

Using properties of determinant, prove that

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

**14.** It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ Rolle's theorem holds in [ 1, 3] with c = $2 + \frac{1}{\sqrt{3}}$. Find the values of 'a' and 'b'

**15.** Determine for what values of x, the function $f(x) = x^3 + \frac{1}{x^3}$ ( $x \neq 0$) is strictly increasing or strictly decreasing

**OR**

Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

**16.** Evaluate $\int_0^2 (x^2 + 3)\, dx$ as limit of sums.

**17.** Find the area of the region bounded by the y-axis, y = cos x and y = sinx, $0 \leq x \leq \frac{\pi}{2}$

**18.** Can $y = ax + \frac{b}{a}$ be a solution of the following differential equation?

$$y = x\frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \quad \text{..............(*)}$$

If no, find the solution of the D.E.(*).

**OR**

Check whether the following differential equation is homogeneous or not

$$x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution y=vx.

**19.** If the vectors $\vec{p} = a\hat{\imath} + \hat{\jmath} + \hat{k}$ , $\vec{q} = \hat{\imath} + b\hat{\jmath} + \hat{k}$ and $\vec{r} = \hat{\imath} + \hat{\jmath} + \widehat{CK}$ are coplanar, then for a, b, c ≠ 1 show that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

**20.** A plane meets the coordinate axes in A, B and C such that the centroid of Δ ABC is the point ( α, β,γ) . Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

**21.** If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.

**22.** The random variable X has a probability distribution P(X) of the following form, where k is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k (ii) Find P(X <2) (iii) Find $P(X \leq 2)$ (iv)Find $P(X \geq 2)$

**23.** A bag contains ( 2n +1) coins. It is known that ‘n’ of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, find the value of ‘n’.

## SECTION-D

**Questions from 24 to 29 are of 6 marks each**

**24.** Using properties of integral, evaluate $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$

**OR**

Find: $\int \frac{\sin x}{\sin^3 x+\cos^3 x}\,dx$

**25.** Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1} 7$$

**OR**

Determine whether the operation $*$ define below on $\mathbb{Q}$ is binary operation or not.

$$a * b = ab+1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in $\mathbb{Q}$ .

**26.** Find the value of x, y and z, if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A′ = $A^{-1}$

**OR**

Verify: A(adj A) = (adj A)A = $|A|\,|\mathrm{I}$ for matrix A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

**27.** Find $\frac{dy}{dx}$, if y = $e^{\sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

**28.** Find the shortest distance between the line x – y + 1 = 0 and the curve $y^2 = x$

**29.** Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:

$\vec{r} = (8 + 3\lambda)\,\hat{\imath} - (9 + 16\lambda)\,\hat{\jmath} + (10 + 7\lambda)\,\hat{k}$

$\vec{r} = 15\,\hat{\imath} + 29\,\hat{\jmath} + 5\,\hat{k} + \mu\,(3\,\hat{\imath} + 8\,\hat{\jmath} - 5\,\hat{k})$

# SAMPLE QUESTION PAPER

## CLASS-XII (2016-17)

## MATHEMATICS (041)

### Marking Scheme

<table>
<tr><td>1.</td><td>$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(-\tan\frac{\pi}{3}\right) = -\frac{\pi}{3}$</td><td>1</td></tr>
<tr><td>2.</td><td>$|3AB| = 3^3\,|A|\,|B| = 27 \times 2 \times 3 = 162$</td><td>1</td></tr>
<tr><td>3.</td><td>Distance of the point (p, q, r) from the x-axis<br>= Distance of the point (p, q, r) from the point (p,0,0)<br>$= \sqrt{q^2 + r^2}$</td><td>1</td></tr>
<tr><td>4.</td><td>gof(x) = g{ f(x)} = g($3x^2 - 5$) = $\frac{3x^2-5}{(3x^2-5)^2+1} = \frac{3x^2-5}{9x^4-30x^2+26}$</td><td>1</td></tr>
<tr><td>5.</td><td>Equivalence relations could be the following:<br>{ (1,1), (2,2), (3,3), (1,2), (2,1)} and (1)<br>{ (1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2)} (1)<br>So, only two equivalence relations.(Ans.)</td><td>2</td></tr>
<tr><td>6.</td><td>AA' = $\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = $I_3$ $\longrightarrow$ (1)<br>because<br>$l_i^2 + m_i^2 + n_i^2 = 1$, for each i = 1, 2, 3 $\longrightarrow$ $1/2$<br>$l_i l_j + m_i m_j + n_i n_j = 0$ ( i ≠j) for each i, j = 1, 2, 3 $\longrightarrow$ $1/2$</td><td>2</td></tr>
<tr><td>7.</td><td>On differentiating $e^y (x + 1) = 1$ w.r.t. x, we get<br>$e^y + (x+1)\, e^y \frac{dy}{dx} = 0$ $\longrightarrow$ (1)<br>$\Rightarrow\ e^y + \frac{dy}{dx} = 0$<br>$\Rightarrow\ \frac{dy}{dx} = -e^y$ $\longrightarrow$ (1)</td><td>2</td></tr>
<tr><td>8.</td><td>Here, $\left\{ \frac{d^2y}{dx^2} + (1 + x) \right\}^3 = -\frac{dy}{dx}$ $\longrightarrow$ (1)<br>Thus, order is 2 and degree is 3. So, the sum is 5 $\longrightarrow$ (1)</td><td>2</td></tr>
<tr><td>9.</td><td>Here, $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$ is same as $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{6}$<br>Cartesian equation of the line is $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$ $\longrightarrow$ (1)<br>Vector equation of the line is<br>$\vec{r} = (-2\,\hat{\imath} + 4\,\hat{\jmath} - 5\,\hat{k}) + \lambda\,(3\,\hat{\imath} + 5\,\hat{\jmath} + 6\,\hat{k})$ $\longrightarrow$ (1)</td><td>2</td></tr>
</table>

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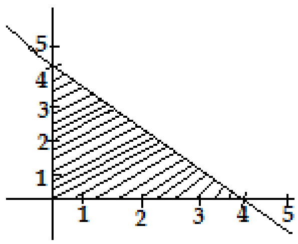
| **10.** | The feasible region is a triangle with vertices<br>O(0,0), A(4,0) and B(0,4)<br>$Z_o = 3\times0 + 4\times0 = 0$<br>$Z_A = 3\times4 + 4\times0 = 12$<br>$Z_B = 3\times0 + 4\times4 = 16$ (1)<br>Thus, maximum of Z is at B(0,4) and the<br>maximum value is 16 $\longrightarrow$ $^1/_2$<br> $\longrightarrow$ $^1/_2$ | 2 |
| **11.** | Sample space = { $B_1B_2$, $B_1G_2$, $G_1B_2$, $G_1G_2$} , $B_1$ and $G_1$ are the older boy and girl respectively.<br>Let $E_1$ = both the children are boys;<br>$E_2$ = one of the children is a boy ;<br>$E_3$ = the older child is a boy<br>Then, (i) $P(E_1/ E_2) = P\left(\frac{E_1\cap E_2}{E_2}\right) = \frac{1/4}{3/4} = \frac{1}{3}$ $\longrightarrow$ (1)<br>(ii) $P(E_1/ E_3) = P\left(\frac{E_1\cap E_3}{E_3}\right) = \frac{1/4}{2/4} = \frac{1}{2}$ $\longrightarrow$ (1) | 2 |
| **12.** | Here, Area(A) = $\frac{\sqrt{3}}{4}x^2$, where 'x' is the side of the equilateral triangle $\longrightarrow$ $^1/_2$<br>So. $\frac{dA}{dt} = \frac{\sqrt{3}}{2} \times \frac{dx}{dt}$ $\longrightarrow$ (1)<br>$= \frac{\sqrt{3}}{2}(10)\,(2) = 10\sqrt{3}$ cm$^2$/sec $\longrightarrow$ $^1/_2$ | 2 |
| **13.** | As $A + B + C = \pi$,<br>$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix}$ $\longrightarrow$ (2)<br>$= 0 \times \begin{vmatrix} 0 & \tan A \\ -\tan A & 0 \end{vmatrix} - \sin B \times \begin{vmatrix} -\sin B & \tan A \\ -\cos C & 0 \end{vmatrix} + \cos C \times \begin{vmatrix} -\sin B & 0 \\ -\cos C & -\tan A \end{vmatrix}$<br>= 0 – sin B tan A cos C + cos C sin B tan A = 0 (Ans.) $\longrightarrow$ (2)<br><br>**OR** | 4 |

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| | Let $\Delta = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$<br>Applying $C_1 \to C_1 + C_3$, we get $\Delta = (a+b+c)\begin{vmatrix} 1 & a-b & a \\ 1 & b-c & b \\ 1 & c-a & c \end{vmatrix}$ $\longrightarrow$ **(1)**<br>Applying $R_2 \to R_2 - R_1$, and $R_3 \to R_3 - R_1$ , we get<br>$\Delta = (a+b+c)\begin{vmatrix} 1 & a-b & a \\ 0 & 2b-a-c & b-a \\ 0 & 2a+b+c & c-a \end{vmatrix}$ $\longrightarrow$ **(1)**<br>Expanding $\Delta$ along first column, we have the result $\longrightarrow$ (2) | 4 |
| **14.** | Since Rolle's theorem holds true, $f(1) = f(3)$<br>i.e., $(1)^3 - 6(1)^2 + a(1) + b = (3)^3 - 6(3)^2 + a(3) + b$<br>i.e., $a + b + 22 = 3a + b$<br>$\Longrightarrow a = 11$ $\longrightarrow$ (2)<br>Also, $f'(x) = 3x^2 - 12x + a$ or $3x^2 - 12x + 11$<br>As $f'(c) = 0$ , we have<br>$3\left(2 + \frac{1}{\sqrt{3}}\right)^2 - 12\left(2 + \frac{1}{\sqrt{3}}\right) + 11 = 0$<br>As it is independent of b, b is arbitrary. $\longrightarrow$ (2) | 4 |
| **15.** | Here, $f'(x) = 3x^2 - 3x^{-4} = \frac{3(x^6-1)}{x^4}$ $\longrightarrow$ (1)<br>$= \frac{3(x^4+x^2+1)}{x^4}(x+1)(x-1)$<br>Critical points are $-1$ and $1$ $\longrightarrow$ (1)<br>$\Longrightarrow$ $f'(x) > 0$ if $x > 1$ or $x < -1$; and $f'(x) < 0$ if $-1 < x < 1$<br>$\{\because \frac{3(x^4+x^2+1)}{x^4}$ always + ive$\}$<br>Hence, f(x) is strictly increasing for $x > 1$ $\longrightarrow$ (1)<br>or $x < -1$; and strictly decreasing for<br>(-1,0)u(0,1) [1] $\longrightarrow$ (1)<br><br>**OR**<br><br>Here, $\frac{dy}{dx} = 3x^2 - 11$ $\longrightarrow$ $^1/_2$<br>So, slope of the tangent is $3x^2 - 11$ | 4<br><br>4 |

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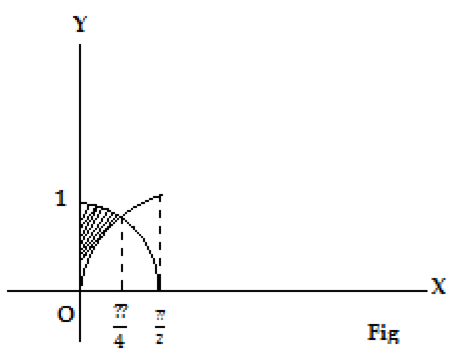
| | Slope of the given tangent line is 1.<br>Thus, $3x^2 - 11 = 1$ → (1)<br>that gives x = $\pm 2$<br>When x = 2, y = 2 – 11 = $-9$<br>When x = – 2, y = $-2 - 11 = -13$<br>Out of the two points (2, – 9) and (–2, –13) → (2)<br>only the point ( 2, – 9) lies on the curve<br>Thus, the required point is ( 2, –9) → $^1/_2$ | |
| **16.** | Here, f(x) = $x^2 + 3$, a = 0 , b = 2 and nh = b – a = 2 → (1)<br>$\int_0^2 (x^2 + 1)\, dx = \lim_{h \to 0} h[\, f(a) + f(a+h) + f(a+2h) + \ldots + f(a + \overline{n-1}\, h)]$ → (1)<br>$= \lim_{h \to 0} h[\, 3 + 1^2h^2 + 3 + 2^2h^2 + 3 + \cdots \ldots + (n-1)^2h^2 + 3]$<br>$= \lim_{h \to 0} h[\, 3n + h^2 \{1^2 + 2^2 + 3^2 + \cdots \ldots (n-1)^2\}]$<br>$= \lim_{h \to 0}[\, 3nh + h^3 \{\frac{(n-1)n\,(2n-1)}{6}\}]$<br>$= \lim_{h \to 0}[\, 3nh + \{\frac{(nh-h)nh\,(2nh-h)}{6}\}]$ → (1)<br>$= \lim_{h \to 0}[\, 3 \times 2 + \{\frac{(2-h)2\,(4-h)}{6}\}]$<br>$= 6 + \frac{16}{6}$ ,i.e., $\frac{26}{3}$ → **(1)** | 4 |
| **17.** | The rough sketch of the bounded region is shown on the right. → (1)<br>Required area = $\int_0^{\pi/4} \cos x\, dx - \int_0^{\pi/4} \sin x\, dx$ → (1)<br>$= (\sin x + \cos x)]_0^{\pi/4}$ → (1)<br>$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos 0$<br>$= \frac{2}{\sqrt{2}} - 1$ , i.e, $(\sqrt{2} - 1)$ sq units → (1)<br><br> | 4 |
| **18.** | y = ax + $\frac{b}{a}$ …. (**1**)<br>gives $\frac{dy}{dx}$ = a → $\left(1\frac{1}{2}\right)$<br>Substituting this value of 'a' in (1), we get | 4 |

$y = x\frac{dy}{dx} + \frac{b}{\frac{dy}{dx}}$ $\longrightarrow$ $\left(1\frac{1}{2}\right)$

**Thus, $y = ax + \frac{b}{a}$ is a solution of the following differential equation $y = x\frac{dy}{dx} + \frac{b}{\frac{dy}{dx}}$.** $\longrightarrow$ **1**

**OR**

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{1+xy+\cos\left(\frac{y}{x}\right)}{x^2} = \frac{y}{x} + \left[\frac{1+\cos\left(\frac{y}{x}\right)}{x^2}\right] \;......(1)$$

Let $F(x,y) = \frac{y}{x} + \left[\frac{1+\cos\left(\frac{y}{x}\right)}{x^2}\right]$.

Then $F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + \left[\frac{1+\cos\left(\frac{\lambda y}{\lambda x}\right)}{(\lambda x)^2}\right]$

$= \frac{y}{x} + \left[\frac{1+\cos\left(\frac{y}{x}\right)}{\lambda^2 x^2}\right] \neq F(x,y)$

Hence, the given D.E. is not a homogeneous equation. $\longrightarrow$ (1)

Putting y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (1), we get

$$v + x\frac{dv}{dx} = v + \frac{1+\cos v}{x^2}$$

$$\Rightarrow \quad \frac{dv}{1+\cos v} = \frac{1}{x^3}\,dx$$

$$\Rightarrow \quad \boldsymbol{sec^2}\left(\frac{v}{2}\right)dv = \frac{2}{x^3}\,dx$$ $\longrightarrow$ (1)

Integrating both sides, we get

$2\tan\frac{v}{2} = -\frac{1}{x^2} + C$ $\longrightarrow$ $1\frac{1}{2}$

or $2\tan\frac{y}{2x} = -\frac{1}{x^2} + C$ $\longrightarrow$ $\frac{1}{2}$

4

**19.** Since the vector $\vec{p}$, $\vec{q}$ and $\vec{r}$ are coplanar

$\therefore [\vec{p}, \vec{q}, \vec{r}] = 0$

$[\vec{p} \quad \vec{q} \quad \vec{r}] = 0$ $\longrightarrow$ (1)

i.e., $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - R_1$ $\longrightarrow$ (1)

or $\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$

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| | $\Rightarrow$ a( b -1)(c-1) -1 (1-a)( c -1) - 1(1-a)( b – 1) = 0<br><br>i.e., a( 1 - b)(1- c) + (1-a)( 1 - c) + (1- a)( 1 – b) = 0 $\longrightarrow$ (1)<br>Dividing both the sides by (1-a)( 1 - b)(1- c), we get<br><br>$\frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$<br>i.e., $-\left(1-\frac{1}{1-a}\right)+\frac{1}{1-b}+\frac{1}{1-c}=0$<br>i.e., $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$ $\longrightarrow$ (1) | |
| **20.** | We know that the equation of the plane having intercepts a, b and c on the three coordinate axes is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ $\longrightarrow$ (1)<br>Here, the coordinates of A, B and C are (a,0,0), (0,b,0) and (0,0,c) respectively.<br>The centroid of $\Delta$ ABC is ( $\frac{a}{3},\frac{b}{3},\frac{c}{3}$ ). $\longrightarrow$ (1)<br>Equating ( $\frac{a}{3},\frac{b}{3},\frac{c}{3}$ ) to ( $\alpha,\beta,\gamma$), we get $a=3\alpha$, $b=3\beta$ and $c=3\gamma$ $\longrightarrow$ (1)<br>Thus, the equation of the plane is $\frac{x}{3\alpha}+\frac{y}{3\beta}+\frac{z}{3\gamma}=1$<br>or $\boldsymbol{\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3}$ $\longrightarrow$ **(1)** | 4 |
| **21.** | Let the distance covered with speed of 25 km/h = x km<br>and the distance covered with speed of 40 km/h = y km (½ )<br>Total distance covered = z km<br>The L.P.P. of the above problem, therefore, is $\longrightarrow$ (1)<br>Maximize z = x + y<br>subject to constraints<br>$4x+5y\le 200$ $\longrightarrow$ (1)<br>$\frac{x}{25}+\frac{y}{40}\le \mathbf{1}$<br>$\mathbf{x\ge 0, y\ge 0}$ **(1)**<br>Any one value $\longrightarrow$ (½ ) | 4 |
| **22.** | Here,<br>X 0 1 2<br>P(X) k 2k 3k<br>(i) Since P(0) + P(1) + P(2)= 1, we have | 4 |

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| | $k + 2k + 3k = 1$<br>i.e., $6k = 1$, or $k = \frac{1}{6}$ $\longrightarrow$ (1)<br>(ii) $P(X<2) = P(0) + P(1) = k + 2k = 3k = \frac{1}{2}$; $\longrightarrow$ (1)<br>(iii) $P(X \leq 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 1$ $\longrightarrow$ (1)<br>(iv) $P(X \geq 2) = P(2) = 3k = \frac{1}{2}$ $\longrightarrow$ (1) | |
| **23.** | Let the events be described as follows:<br>$E_1$ : a coin having head on both sides is selected.<br>$E_2$ : a fair coin is selected.<br>A : head comes up in tossing a selected coin<br>$P(E_1) = \frac{n}{2n+1}$; $P(E_2) = \frac{n+1}{2n+1}$; $P(A/E_1) = 1$; $P(A/E_2) = \frac{1}{2}$ $\longrightarrow$ (2)<br>It is given that $P(A) = \frac{31}{42}$. So,<br>$P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) = \frac{31}{42}$<br>$\Rightarrow \quad \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2} = \frac{31}{42}$ $\longrightarrow$ (1)<br>$\Rightarrow \quad \frac{1}{2n+1}\left[n + \frac{n+1}{2}\right] = \frac{31}{42}$<br>$\Rightarrow \quad 42(3n+1) = 62(2n+1)$<br>$\Rightarrow \quad 2n = 20,\ or\ n = 10$ $\longrightarrow$ (1) | 4 |
| **24.** | $I = \int_0^{\pi} \frac{x}{1+\sin x}\,dx = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}\,dx$ (1)<br>$= \pi \int_0^{\pi} \frac{1}{1+\sin x}\,dx - \int_0^{\pi} \frac{x}{1+\sin x}\,dx$<br>$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{1}{1+\sin x}\,dx$ (1)<br>$\Rightarrow \quad \frac{\pi}{2} \int_0^{\pi} \frac{1}{1+\cos\left(\frac{\pi}{2} - x\right)}\,dx$<br>$\Rightarrow \quad \frac{\pi}{2} \int_0^{\pi} \frac{1}{2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}\,dx$<br>$\Rightarrow \quad \frac{\pi}{4} \int_0^{\pi} \sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)\,dx$ (1)<br>$\Rightarrow \quad I = \frac{\pi}{4}\left[-2\tan\left[\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]\right]_0^{\pi}$ (2)<br>$\Rightarrow \quad I = \frac{\pi}{4}\left[2 - (-2)\right] = \pi$ (1)<br>**OR** | 6 |

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| | Let I = $\int \frac{\sin x}{\sin^3 x+\cos^3 x}\,dx = \int \frac{\tan x \sec^2 x}{\tan^3 x+1}\,dx$ (½)<br>On substituting tan x = t and $\sec^2 x\,dx = dt$, we get (1)<br>$I = \int \frac{t}{t^3+1}\,dt = \int \frac{t}{(t+1)(t^2-t+1)}\,dt$ (½)<br>$= -\frac{1}{3}\int \frac{1}{t+1}\,dt + \frac{1}{3}\int \frac{t+1}{t^2-t+1}\,dt$<br>$= -\frac{1}{3}\log\lvert t+1\rvert + \frac{1}{6}\int \frac{(2t-1)+3}{t^2-t+1}\,dt$ (1)<br>$= -\frac{1}{3}\log\lvert t+1\rvert + \frac{1}{6}\int \frac{2t-1}{t^2-t+1}\,dt + \frac{1}{2}\int \frac{1}{t^2-t+1}\,dt$<br>$= -\frac{1}{3}\log\lvert t+1\rvert + \frac{1}{6}\log\lvert t^2-t+1\rvert + \frac{1}{2}\int \frac{1}{\left(t-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dt$<br>$= -\frac{1}{3}\log\lvert t+1\rvert + \frac{1}{6}\log\lvert t^2-t+1\rvert + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)$ (2)<br>$= -\frac{1}{3}\log\lvert \tan x+1\rvert + \frac{1}{6}\log\lvert \tan^2 x-\tan x+1\rvert + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right) + c$ (1) | 6 |
| **25.** | $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1}7$<br>$\Longrightarrow \tan^{-1}\left[\frac{\left(\frac{x+1}{x-1}\right)+\left(\frac{x-1}{x}\right)}{1-\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right] = -\tan^{-1}7$ , if $\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) < 1$ .....(*) (2)<br>$\Longrightarrow \tan^{-1}\left[\frac{x(x+1)+(x-1)^2}{(x-1)x-(x+1)(x-1)}\right] = -\tan^{-1}7$<br>$\Longrightarrow \frac{(x^2+x)+(x^2+1-2x)}{(x^2-x)-(x^2-1)} = \tan\left[-\tan^{-1}7\right]$<br>$\Longrightarrow \frac{2x^2-x+1}{-x+1} = -7$ (1)<br>$\Longrightarrow 2x^2-8x+8 = 0$<br>$\Longrightarrow (x-2)^2 = 0$<br>$\Longrightarrow x = 2$ (1)<br>Let us now verify whether x = 2 satisfies the condition (*)<br>For x = 2,<br>$\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) = 3\times\frac{1}{2} = \frac{3}{2}$ which is not less than 1<br>Hence, this value does not satisfy the condition (*) (1)<br>i.e., there is no solution to the given trigonometric equation. (1)<br>**OR**<br>Given $*$ on $\mathbb{Q}$, defined by a$*$b = ab+1<br>Let, a$\in\mathbb{Q}$, b$\in\mathbb{Q}$ then<br>ab$\in\mathbb{Q}$ | 6 |

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and (ab+1)∈Q

$\Rightarrow$ a∗b=ab+1 is defined on Q

$\therefore$ ∗ is a binary operation on Q (1)

**Commutative**: a∗b = ab+1

b∗a = ba+1

=ab+1 ($\because$ ba = ab in Q)

$\Rightarrow$ a∗b =b∗a

So ∗ is commutative on Q (1)

**Associative**: (a∗b)∗c= (ab+1)∗c =(ab+1)c+1

= abc+c+1

a∗(b∗c)=a∗(bc+1)

= a(bc+1)+1

= abc+a+1

$\therefore$ (a∗b)∗c $\neq$ a∗(b∗c)

So ∗ is not associative on $\mathbb{Q}$ (1)

**Identity Element :** Let e ∈$\mathbb{Q}$ be the identity element, then for every a ∈$\mathbb{Q}$

a∗e=a and e∗a=a

ae+1=a and ea+1=a

$\Rightarrow$ e= $\frac{a-1}{a}$ and e= $\frac{a-1}{a}$ (1)

e is not unique as it depend on \`a' ,hence identity element does not exist for ∗ (1)

**Inverse**: since there is no identity element hence, there is no inverse. (1)

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**26.** The relation A' = $A^{-1}$ gives A'A = $A^{-1}A$ = $I$ (1)

Thus, $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\left(1\frac{1}{2}\right)$

$$\Rightarrow \begin{bmatrix} 0 + x^2 + x^2 & 0 + xy - xy & 0 - xz + xz \\ 0 + xy - xy & 4y^2 + y^2 + y^2 & 2yz - yz - yz \\ 0 - zx + zx & 2yz - yz - yz & z^2 + z^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (2)

$\Rightarrow\ 2x^2 = 1;\ 6y^2 = 1$ and $3z^2 = 1$

$\Rightarrow\ \mathrm{x} = \pm\frac{1}{\sqrt{2}};\quad \mathrm{y} = \pm\frac{1}{\sqrt{6}};\quad \mathrm{z} = \pm\frac{1}{\sqrt{3}}$ $\left(1\frac{1}{2}\right)$

**OR**

| | | |
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| | Here, $|A| = \begin{vmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{vmatrix}$ = 1(0+0) +1(9+2) +2(0– 0) =11 (1)<br><br>$\implies |A|\mathrm{I} = \begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$ ...................(1) (½)<br><br>$\text{adj A} = \begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix}$ (2)<br><br>Now, A(adj A) = $\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}\begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix} = \begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$ (1)<br><br>and (adj A)A = $\begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix} = \begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$ (1)<br><br>Thus, it is verified that A(adj A) = (adj A)A = $|A|\mathrm{I}$ (½) | 6 |
| **27.** | Putting x = cos 2Θ in $\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$, we get (1)<br><br>$2\tan^{-1}\sqrt{\frac{1-cos\,2\Theta}{1+cos\,2\Theta}}$<br><br>i.e., $2\tan^{-1}\sqrt{\frac{2sin^2\Theta}{2cos^2\Theta}} = 2\tan^{-1}(\tan\Theta) = 2\Theta = \cos^{-1}x$ (2)<br><br>Hence, y = $e^{\sin^2 x}\cos^{-1}x$<br><br>$\implies \log y = \sin^2 x + \log(\cos^{-1}x)$<br><br>$\implies \frac{1}{y}\times\frac{dy}{dx} = 2\sin x\cos x + \frac{1}{\cos^{-1}x}\times\frac{-1}{\sqrt{1-x^2}} = \sin 2x - \frac{1}{\cos^{-1}x\sqrt{1-x^2}}$ (2)<br><br>$\implies \frac{dy}{dx} = e^{\sin^2 x}\cos^{-1}x\left[\sin 2x - \frac{1}{\cos^{-1}x\sqrt{1-x^2}}\right]$ (1) | 6 |
| **28.** | Let $(t^2, t)$ be any point on the curve $y^2 = x$. Its distance (S) from the<br><br>line x – y + 1 = 0 is given by $1/2$<br><br>S = $\left\|\frac{t-t^2-1}{\sqrt{1+1}}\right\|$ $1/2$<br><br>$= \frac{t^2-t+1}{\sqrt{2}}$ $\{\because t^2 - t + 1 = \left(t-\frac{1}{2}\right)^2 + \frac{3}{4} > 0\}$ (1)<br><br>$\implies \frac{dS}{dt} = \frac{1}{\sqrt{2}}(2t-1)$ (1)<br><br>and $\frac{d^2S}{dt^2} = \sqrt{2} > 0$ (1)<br><br>Now, $\frac{dS}{dt} = 0 \implies \frac{1}{\sqrt{2}}(2t-1) = 0$ ,i.e., t = $\frac{1}{2}$ (1)<br><br>Thus, S is minimum at $t = \frac{1}{2}$ | 6 |

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So, the required shortest distance is $\frac{(\frac{1}{2})^2-(\frac{1}{2})+1}{\sqrt{2}} = \frac{3}{4\sqrt{2}}$, or $\frac{3\sqrt{2}}{8}$ (1)

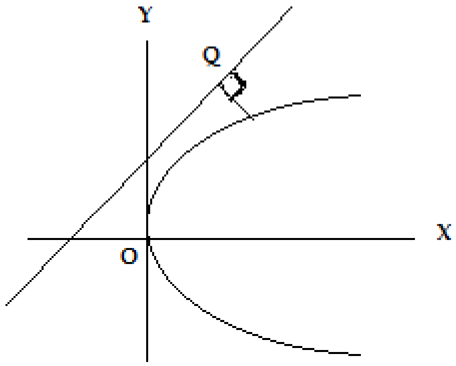


Fig. 1

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1) the line which are neither intersecting nor parallel. (1)

2) The given equations are

$\vec{r} = 8\,\hat{\imath} - 9\,\hat{\jmath} + 10\,\hat{k} + \mu\,(3\,\hat{\imath} - 16\,\hat{\jmath} + 7\,\hat{k})$ ....................(1) (½)

$\vec{r} = 15\,\hat{\imath} + 29\,\hat{\jmath} + 5\,\hat{k} + \mu\,(3\,\hat{\imath} + 8\,\hat{\jmath} - 5\,\hat{k})$ ....................(2)

Here, $\overrightarrow{a_1} = 8\,\hat{\imath} - 9\,\hat{\jmath} + 10\,\hat{k}$; $\overrightarrow{a_2} = 15\,\hat{\imath} + 29\,\hat{\jmath} + 5\,\hat{k}$

$\overrightarrow{b_1} = 3\,\hat{\imath} - 16\,\hat{\jmath} + 7\,\hat{k}$; $\overrightarrow{b_2} = 3\,\hat{\imath} + 8\,\hat{\jmath} - 5\,\hat{k}$

Now, $\overrightarrow{a_2} - \overrightarrow{a_1} = (15-8)\,\hat{\imath} + (29+9)\,\hat{\jmath} + (5-10)\,\hat{k} = 7\,\hat{\imath} + 38\,\hat{\jmath} - 5\,\hat{k}$ (½)

and

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\,\hat{\imath} + 36\,\hat{\jmath} + 72\,\hat{k}$$ (1)

$\Rightarrow (\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1}) = (24\,\hat{\imath} + 36\,\hat{\jmath} + 72\,\hat{k}).(7\,\hat{\imath} + 38\,\hat{\jmath} - 5\,\hat{k}) = 1176$ (1)

Shortest distance = $\left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$ (1)

$= \left|\frac{1176}{\sqrt{24^2+36^2+72^2}}\right| = \frac{1176}{\sqrt{7056}} = \frac{1176}{84} = \frac{98}{7}$ (1)

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